Statistical models and methods for analyzing point processes on linear networks or with linear structures

This PhD course consists of 5 lectures, each of no more than 90 minutes. Their dates are Tuesdays and Thursdays (22.,24.,29.,31.10.) at 2 p.m. and Friday 25.10. at 10:40 a.m.

The 3 first lectures are mainly based on the articles [1, 3, 4] and partly on the articles [2, 5, 6]: We develop statistical models for spatial point processes with a linear structure and we develop functional summary statistics called the cylindrical K-function and space-sphere K-functions. One model is the Poisson line cluster point process where first a Poisson line process is generated, second Poisson point processes are independently generated on the lines, and third the points of these are randomly disturbed; another model is based on a determinantal point process so that the lines tend to be more separated. Furthermore, the functional summary statistics can be used for both parameter estimation and model checking. The models and methods are applied for analyzing point patterns on dendrite networks in order to investigate the minicolumn hypothesis in neuroscience which claims that brain cells are organized in columnar structures; here the points represent the centres of pyramidal cells (or neurons); and for one dataset the direction from the centre to the apex of each cell is also given – this is then viewed as a marked point pattern, where we test for independence between points and directions.

The 2 last lectures are based on the articles [7, 8]: We consider point processes on linear networks or more generally graphs and their edges, i.e., graphs with edges viewed as line segments or more general sets with a coordinate system allowing us to consider points on the graph which are vertices or points on an edge – one example is a dendrite network. Due to the special geometric structure of linear networks it is a challenge to develop point process models and functional summary statistics. For example, Ripley's K-function as defined for homogeneous point processes in space can not immediately be adapted to linear networks. Moreover, only a few point process models have so far been developed. For the analysis of spine locations on dendrite trees, we study a new model where an inhomogeneous Poisson process is dependently thinned; in brief, the retention probability is specified via a Gaussian process with an isotropic covariance function; here, isotropy refers to that the covariance at two locations only depends on the distance between the locations. However, it is far from trivial to specify what is meant by distance (we study two metrics: the geodesic distance and the resistance metric) and to establish a theory for isotropic covariance functions on graphs and their edges – this will indeed be the most technical part of the course.

[1] J. Møller, F. Safavimanesh and J.G. Rasmussen (2016). The cylindrical K-function and Poisson line cluster point processes. *Biometrika*, 103, 937-954.

[2] A.H. Rafati, F. Safavimanesh, K.-A. Dorph-Petersen, J.G. Rasmussen, J. Møller and J.R. Nyengaard (2016). Detection and spatial characterization of minicolumnarity in the human cerebral cortex. *Journal of Microscopy*, 261, 115-126.

[3] J. Møller, H.S. Christensen, F. Cuevas-Pacheco and A.D. Christoffersen (2019). Structured space-sphere point processes and K-functions. To appear in *Methodology and Computing in Applied Probability*. Available at arXiv:1812.08986.

[4] A.D. Christoffersen, J. Møller and Heidi Christensen (2019). Modelling columnarity of pyramidal cells in the human cerebral cortex based on directionally marked 3D point pattern datasets. In preparation.

[5] F. Lavancier, J. Møller and E. Rubak (2015). Determinantal point process models and statistical inference. *Journal of Royal Statistical Society: Series B (Statistical Methodology)*, 77, 853-877.

[6] J. Møller and A.D. Christoffersen (2018). Pair correlation functions and limiting distributions of iterated cluster point processes. *Journal of Applied Probability*, 55, 789-809. DOI:10.1017/jpr.2018.50. Available at arXiv:1711.08984.

[7] E. Anderes, J. Møller and J.G. Rasmussen (2017). Isotropic covariance functions on graphs and their edges. Submitted for journal publication. Research Report 10, 2017, Centre for Stochastic Geometry and Advanced Bioimaging. Available at arXiv:1710.01295.

[8] H.S. Christensen and J. Møller (2019). Modelling spine locations on dendrite trees with inhomogeneous Cox point processes. In preparation.